Project II: Optimizing US-global Health Commodity Shipment Plan

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Abstract

Efficient and sustainable distribution of health commodities is critical for global health initiatives. This study examines two optimization strategies within the U.S. government's Global Health Commodity Support Plan: a single-trip shipment minimizing total travel distance and a multi-phase approach utilizing warehouses near JFK and LAX airports. The single-trip plan emphasizes simplicity and resource constraints, while the phased strategy explores trade-offs between distributed and centralized operations. Results highlight the effectiveness of distributed shipments in reducing environmental impact and improving efficiency, while centralized strategies offer logistical simplicity. These findings support strategic route planning to enhance supply chain sustainability and adaptability.

1 Introduction

The efficient and sustainable distribution of essential health commodities is a critical priority for global health initiatives. As part of its Global Health Commodity Support Plan, the U.S. government provides vital support by shipping antiretroviral (ARV) medications and HIV lab supplies to countries experiencing shortages. This initiative plays a pivotal role in addressing global health challenges, but its success hinges on the design of a cost-effective and environmentally conscious supply chain.

This report focuses on two key optimization challenges within this initiative. The first involves designing a single-trip shipment plan to deliver all health commodities to supported countries in one consolidated route. The shipment begins at JFK Airport and ends at LAX Airport, encompassing all destinations in between. The goal is to minimize the total shipping distance, which is proportional to the haversine distances between destinations, thereby reducing CO2 emissions. This scenario assumes no intermediate warehouse support and prioritizes efficiency and sustainability for a single comprehensive trip.

The second challenge explores a multi-phase shipment approach, leveraging two warehouses located near JFK and LAX airports. In this scenario, shipments are distributed in three distinct phases, with each plane departing from and returning to its original airport after completing its delivery route. The objective is to minimize the total shipping distance for each phase, ensuring efficiency while adhering to environmental sustainability goals. By introducing multiple phases and warehouse support, this approach aims to optimize resource utilization and reduce logistical complexity.

Through sensitivity analysis and optimization techniques, this report seeks to provide actionable recommendations for each scenario. By addressing these challenges, the U.S. government can enhance the sustainability and efficiency of its Global Health Commodity Support Plan, delivering life-saving supplies to countries in need while minimizing environmental impact.

2 The Problem of Optimizing Shipment

2.1 Set and Indices

- $A \in$ Airport_code = {JFK, LAX, ABJ, SGN, ..., TIP, BZE}
- $i, j \in A$: Indicate each airport code

2.2 Parameters

- $n = 45$: Total number of airports in the dataset
- $D_{i,j}$: The distance between airports i and j, such that

$$D_{i,j} = 2r \times arcsin(\sqrt{sin^2(\frac{\varphi_j - \varphi_i}{2}) + cos(\phi_i) \times cos(\phi_j) \times sin^2(\frac{\lambda_j - \lambda_i}{2})})$$

  where

  - $r \approx 3958.756\ miles$ (Earth's radius).
  - ϕ_i, ϕ_j : The latitudes of airports i and j.
  - λ_i, λ_j : The longitudes of airports i and j.

- D : A distance matrix that is used to store the distance between airports i and j, with form:

$$D = \begin{bmatrix} D_{JFK,JFK} & D_{JFK,LAX} & D_{JFK,ABJ} & \cdots & D_{JFK,BZE} \\ D_{LAX,JFK} & D_{LAX,LAX} & D_{LAX,ABJ} & \cdots & D_{LAX,BZE} \\ D_{ABJ,JFK} & D_{ABJ,LAX} & D_{ABJ,ABJ} & \cdots & D_{ABJ,BZE} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ D_{BZE,JFK} & D_{BZE,LAX} & D_{BZE,ABJ} & \cdots & D_{BZE,BZE} \end{bmatrix}$$

  where

  - $D_{i,j} = D_{j,i}$
  - $D_{i,i} = 0$

2.3 Decision Variables

- $x_{ij} \in \{0, 1\}$: The binary variable that indicates the flight from airport i to j.

$$x_{i,j} = \begin{cases} 1, & \text{if the flight departs from airport } i \text{ and arrives at airport } j, \\ 0, & \text{otherwise.} \end{cases}$$

- $u_i \in \mathbb{Z}^+$: The "potential" value that indicates the order of the corresponding airport i in tour.

2.4 Objective Function

- Minimize the total length of the shipping route to deliver all health commodities to the supported countries.

$$Min \sum_{i \in A} \sum_{j \in A, i \neq j} D_{i,j} \times x_{i,j}$$

2.5 Constraints

1. **Start Constraint:** The route will start at JFK, which means depart from JFK exactly once

   $$\sum_{j \in A, j \neq i} x_{i,j} = 1$$

   where $i = JFK, j \neq JKF$

2. **Return Constraint:** The flight will return to LAX, which means arrive at LAX exactly once.

   $$\sum_{i \in A, i \neq j} x_{i,j} = 1$$

   where $j = LAX, i \neq LAX$

3. **Departure Constraint:** The flight will only depart from each airport once.

   $$\sum_{j \in A, j \neq i} x_{i,j} = 1$$

   such that $\forall i \in A, i \neq JFK, LAX$

4. **Arrival Constraint:** The flight will only arrive at each airport once.

   $$\sum_{j \in A, j \neq i} x_{j,i} = 1$$

   such that $\forall i \in A, i \neq JFK, LAX$

5. **Miller-Tucker-Zemlin SubtourElimination Constraint:** Ensure there is no subtour in the route.

   $$u_i - u_j + n \times x_{i,j} \leq n - 1$$

   such that $\forall i, j \in A, i \neq j$

2.6 Results

The optimization of shipping routes was performed to minimize the total travel distance across all phases of the U.S. government's Global Health Commodity Support Plan. Using a computational approach based on the haversine distances between destinations, the most efficient routes for delivering health commodities in each phase were identified.

The optimized shipping route for the first phase was determined to follow this sequence:

- JFK → TIP → BEY → KBL → ISB → FRU → ALA → SGN → ADD → KRT → JUB → EBB → KGL → BJM → NBO → DAR → LLW → LUN → HRE → MPM → SHO → MSU → JNB → GBE → WDH → LAD → FIH → DLA → LOS → COO → LFW → ACC → ABJ → OUA → BKO → ROB → FNA → CKY → DSS → GEO → JBQ → PAP → BZE → GUA → LAX

The total distance traveled for this optimized route was calculated to be **35311 miles**, minimizing the overall travel distance compared to unoptimized routes.

The visualization (Figure 1) clearly illustrates this route, highlighting the sequential delivery plan across all targeted destinations before returning to the departure point.

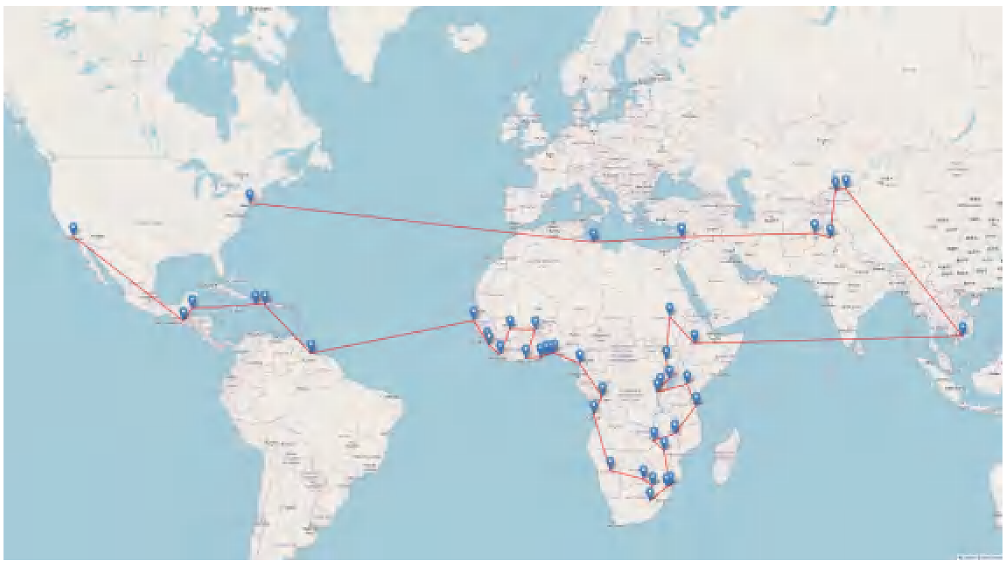


Figure 1: Visualization of the optimized shipping routes.

These results demonstrate that the optimization method effectively supports the U.S. government's goals for cost-effective and sustainable delivery of health commodities to the supported countries. The derived route sequence and visualizations serve as a valuable reference for planning and implementation.

3 The Problem of Optimizing Splitted Shipment

3.1 Set and Indices

We use the same set and indices as in "The Problem of Optimizing Shipment" in **Section 2.1**.

3.2 Parameters

We use the same parameters as "The Problem of Optimizing Shipment" in **Section 2.2**.

3.3 Decision Variables

- $x_{ij} \in \{0,1\}$: The binary variable that indicate whether the first flight travels from airport i to airport j.

$$x_{i,j} = \begin{cases} 1, & \text{if the first flight departs from airport } i \text{ and arrives at airport } j, \\ 0, & \text{otherwise.} \end{cases}$$

- $y_{ij} \in \{0,1\}$: The binary variable that indicate whether the second flight travels from airport i to airport j.

$$y_{i,j} = \begin{cases} 1, & \text{if the second flight departs from airport } i \text{ and arrives at airport } j, \\ 0, & \text{otherwise.} \end{cases}$$

- $u_i \in \mathbb{Z}^+$: The "potential" value for first flight that indicates the order of the corresponding airport i in tour.
- $w_i \in \mathbb{Z}^+$: The "potential" value for second flight that indicates the order of the corresponding airport i in the tour.

3.4 Objective Function

- Minimize the total shipping route length for first and second flights to deliver all health commodities to the supported countries.

$$Min(\sum_{i\in A}\sum_{j\in A} D_{i,j} \times x_{i,j} \ + \sum_{i\in A}\sum_{j\in A} D_{i,j} \times y_{i,j})$$

3.5 Constraints

3.5.1 Case 1

In Case 1, two flights must be used for commodity transportation.

1. **Separate Tour Constraint:** Ensure each airport will be visited exactly once by one of the flights. That means each airport can be visited either by first flight or second flight, but not both.

$$\sum_{j\in A, j\neq i} x_{i,j} \ + \sum_{j\in A, j\neq i} y_{i,j} = 1$$

where $\forall i \in A, i \neq JFK, LAX$

2. **Start Constraint:**

   - **For First Flight:** The first flight will start at JFK. That means it will depart from JFK exactly once.

$$\sum_{j\in A, j\neq JFK} x_{JFK,j} = 1$$

- **For Second Flight:** The second flight will start at LAX. That means it will depart from LAX exactly once.

$$\sum_{j \in A, j \neq LAX} y_{LAX,j} = 1$$

3. **Return Constraint:**

- **For First Flight:** The first flight will return to JFK, which means arrive at JFK exactly once.

$$\sum_{i \in A, i \neq JFK} x_{i,JFK} = 1$$

- **For Second Flight:** The second flight will return to LAX, which means arrive at LAX exactly once.

$$\sum_{i \in A, i \neq LAX} y_{i,LAX} = 1$$

4. **Navigation Restriction Constraint:**

- **For First Flight:** Ensure the first flight will not arrive to or depart from LAX.

$$\sum_{i \in A} x_{i,LAX} = 0, \ \sum_{j \in A} x_{LAX,j} = 0$$

- **For Second Flight:** Ensure the second flight will not arrive to or depart from JFK.

$$\sum_{i \in A} y_{i,JFK} = 0, \ \sum_{j \in A} y_{JFK,j} = 0$$

5. **Navigation Conservation Constraint:**

- **For First Flight:** During the tour, the first flight need to arrive at one airport exactly once and depart from the same airport exactly once.

$$\sum_{j \in A, j \neq i} x_{i,j} = \sum_{j \in A, j \neq i} x_{j,i}$$

where $\forall i \in A, i \neq JFK$

- **For Second Flight:** During the tour, the second flight need to arrive at one airport exactly once and depart from the same airport exactly once.

$$\sum_{j \in A, j \neq i} y_{i,j} = \sum_{j \in A, j \neq i} y_{j,i}$$

where $\forall i \in A, i \neq LAX$

6. **Miller-Tucker-Zemlin SubtourElimination Constraint:** Ensure there is no subtour in the route for first flight and second flight.

- **For First Flight:**
$$u_i - u_j + 1 \le (n-1) \cdot (1 - x_{i,j})$$
where $\forall i, j \in A, i \neq j, i \neq JFK$
- **For Second Flight:**
$$w_i - w_j + 1 \le (n-1) \cdot (1 - y_{i,j})$$
where $\forall i, j \in A, i \neq j, i \neq LAX$

3.5.2 Case 2

In Case 2, both flights may not be used exclusively for commodity transportation. This means that flights departing from JFK or flights departing from LAX can be canceled. We implement the restrictions by modifying the Start and Return Constraints and adding a Balance Constraint, leaving the rest of the constraints the same as in Case 1 in **Section 3.5.1**.

1. **Start Constraint:**
   - **Situation 1:** The first flight must depart from JFK exactly once, and the second flight may depart from LAX or not. This means that the second flight departing from LAX can be canceled.
   $$\sum_{j \in A, j \neq JFK} x_{JFK,j} = 1, \sum_{j \in A, j \neq LAX} y_{LAX,j} \le 1$$
   - **Situation 2:** The second flight must depart from LAX exactly once, and the first flight may depart from JFK or not. This means that the first flight departing from JFK can be canceled.
   $$\sum_{j \in A, j \neq JFK} x_{JFK,j} \le 1, \sum_{j \in A, j \neq LAX} y_{LAX,j} = 1$$
2. **Return Constraint:**
   - **Situation 1:** The first flight must arrive at JFK exactly once, and the second flight may arrive at LAX or not.
   $$\sum_{i \in A, i \neq JFK} x_{i,JFK} = 1, \sum_{i \in A, i \neq LAX} y_{i,LAX} \le 1$$
   - **Situation 2:** The second flight must arrive at LAX exactly once, and the first flight may arrive at JFK or not.
   $$\sum_{i \in A, i \neq JFK} x_{i,JFK} \le 1, \sum_{i \in A, i \neq LAX} y_{i,LAX} = 1$$
3. **Balance Constraint:**

- **Situation 1:** Ensure that the number of times the second flight departs from LAX equals the number of times it returns to LAX.

$$\sum_{i \in A, i \neq LAX} y_{i,LAX} - \sum_{j \in A, j \neq LAX} y_{LAX,j} = 0$$

- **Situation 2:** Ensure that the number of times the first flight departs from JFK equals the number of times it returns to JFK.

$$\sum_{i \in A, i \neq JFK} x_{i,JFK} - \sum_{j \in A, j \neq JFK} x_{JFK,j} = 0$$

3.6 Results

The optimization of the problem of optimizing split shipping was performed in two distinct cases. The results for each case are presented below, along with visualizations of the optimized shipping routes.

3.6.1 Case 1

For Case 1, the optimized shipping routes were determined as follows:

- **Route from JFK:** JFK → TIP → BEY → KBL → ISB → FRU → ALA → SGN → ADD → KRT → JUB → EBB → KGL → BJM → NBO → DAR → LLW → LUN → HRE → MPM → SHO → MSU → JNB → GBE → WDH → LAD → FIH → DLA → LOS → COO → LFW → ACC → ABJ → OUA → BKO → ROB → FNA → CKY → DSS → GEO → JBQ → PAP → JFK.
- **Route from LAX:** LAX → BZE → GUA → LAX.

The total mileage for Case 1 is **37960 miles**. The visualization of these routes is presented in Figure 2.

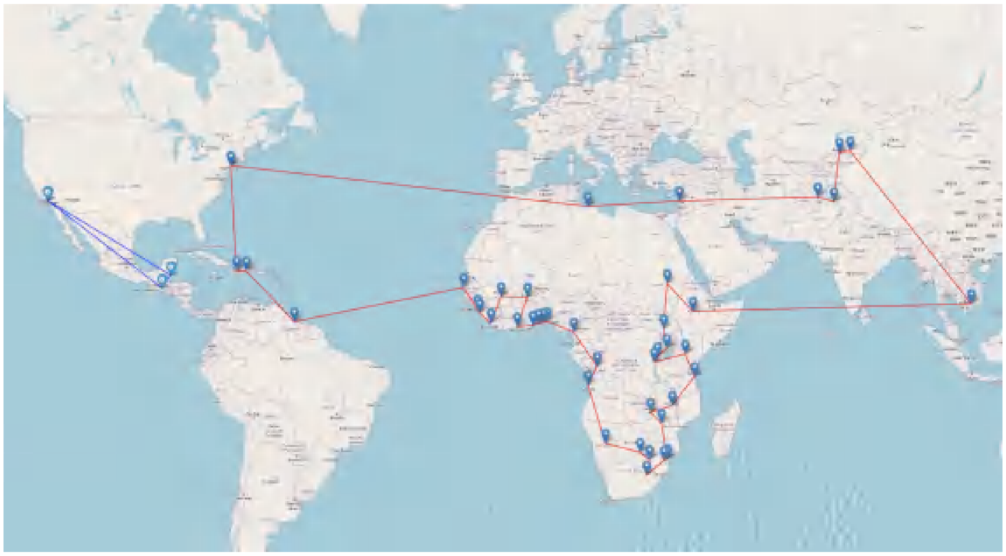


Figure 2: Optimized shipping routes for Case 1.

3.6.2 Case 2

In Case 2 we discussed two situations: 1.)the first flight departing from JFK can be canceled and 2.)the second flight departing from LAX can be canceled. By comparison, we found that the situation of canceling second flight from LAX had a shorter flight distance, so we took this result as the final result of Case 2 in the report and discussed it in depth in the following content. The optimized shipping route was determined as follows:

- JFK → BZE → GUA → PAP → JBQ → GEO → DSS → CKY → FNA → ROB → BKO → OUA → ABJ → ACC → LFW → COO → LOS → DLA → FIH → LAD → WDH → GBE → JNB → MSU → SHO → MPM → HRE → LUN → LLW → DAR → NBO → BJM → KGL → EBB → JUB → KRT → ADD → SGN → ALA → FRU → ISB → KBL → BEY → TIP → JFK.

The total mileage for Case 2 is **35116 miles**. The visualization of this route is presented in Figure 3.

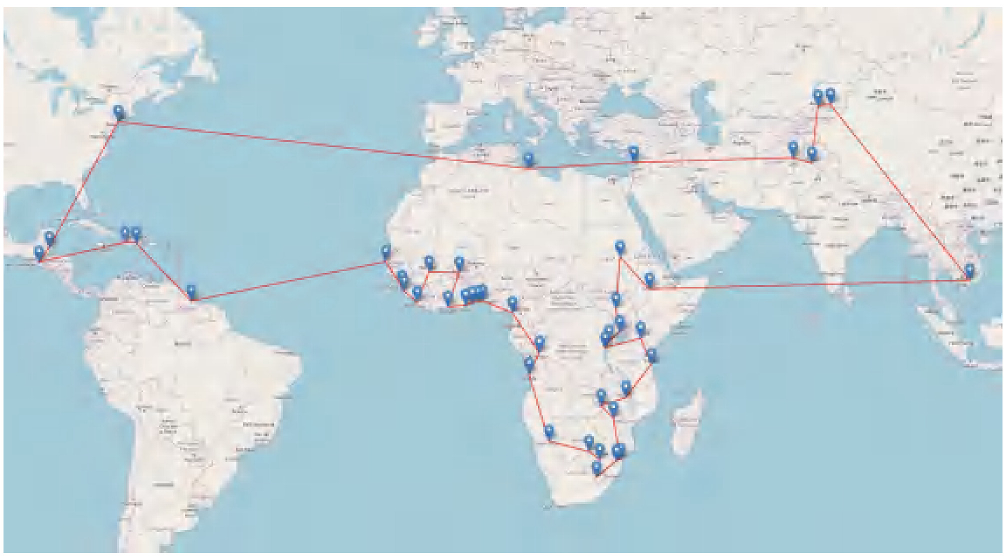


Figure 3: Optimized shipping route for Case 2.

3.6.3 Comparison of Cases

In Case 1, two separate routes were optimized for the departure points JFK and LAX. This led to a clear division of destinations between the two hubs, which helps reduce the operational complexity. However, the total mileage for this approach is **37960 miles**, which is higher than Case 2.

In Case 2, all destinations were served from JFK, resulting in a single continuous route. This approach reduced the total mileage to **35116 miles**, making it more efficient in terms of travel distance. However, consolidating shipments at a single hub could pose operational challenges, such as increased scheduling complexity.

These results demonstrate a trade-off between operational simplicity and mileage efficiency. Case 1 provides a split-shipment strategy that simplifies logistics, while Case 2 achieves lower total mileage by consolidating all shipments into a single hub.

4 Sensitivity Analysis

To evaluate the robustness of the optimized routes under variations in airport coordinates, we conducted a sensitivity analysis. The analysis involved perturbing the original latitude and longitude data of the airports with Gaussian random noise. The random noise had a mean of 0 and standard deviations of 1, 2, 5, and 10. For each scenario, the changes in the total distance and the resulting routes were observed.

4.1 Experimental Design

The sensitivity analysis was conducted for all three cases:

1. **Problem of Optimizing Shipment:** The single consolidated route (Problem 1).
2. **Problem of Optimizing Splitted Shipment - Case 1:** Distributed shipments via two hubs (JFK and LAX).
3. **Problem of Optimizing Splitted Shipment - Case 2:** Centralized shipment with a single hub (JFK).

For each case, four levels of perturbation (standard deviations of 1, 2, 5, and 10) were applied, resulting in a total of 12 experiments. The optimized routes and distances for each scenario were visualized and compared to the original optimal solution.

4.2 Results

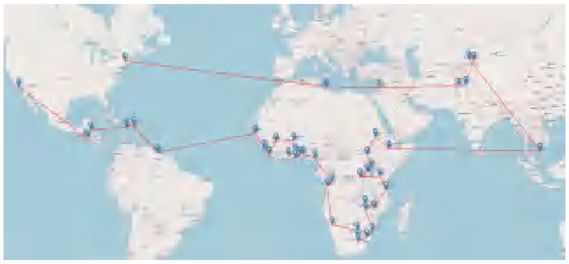
4.2.1 Problem of Optimizing Shipment

The total distances for the single consolidated route under varying levels of perturbation are summarized in Table 1. As the standard deviation of the noise increases, the total distance gradually deviates from the original optimal solution.

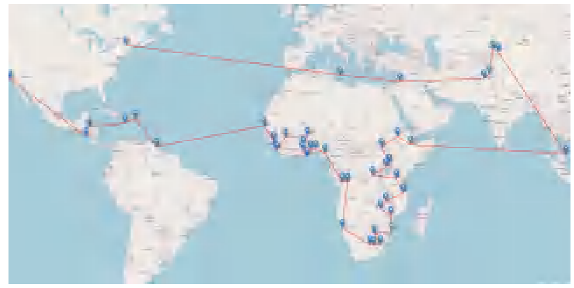
Standard Deviation (std)	Total Distance (miles)
1	35,302.4
2	35,342.1
5	37,432.8
10	43,114.5

Table 1: Sensitivity analysis for Problem of Optimizing Shipment.

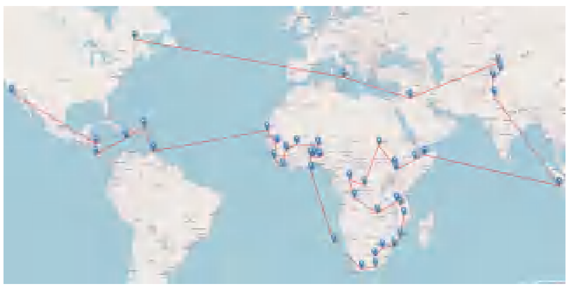
The following visualizations (Figure 4) illustrate the optimized routes under different levels of perturbation for Problem 1.



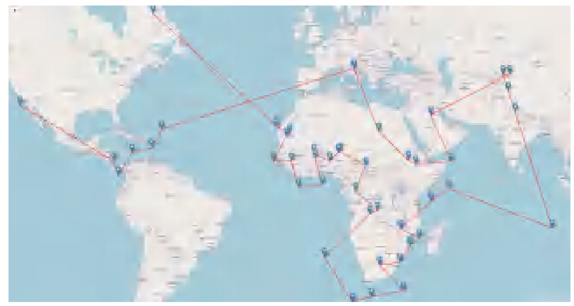
(a) Standard deviation: 1



(b) Standard deviation: 2



(c) Standard deviation: 5



(d) Standard deviation: 10

Figure 4: Optimized routes under different perturbations for Problem of Optimizing Shipment.

4.2.2 Problem of Optimizing Splitted Shipment

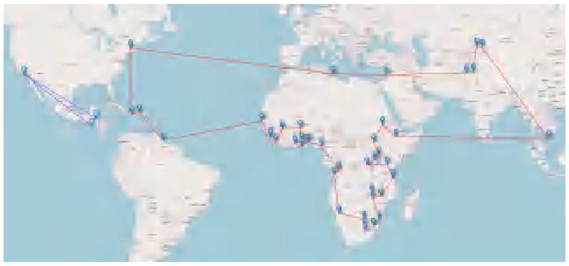
Case 1: Distributed Shipments

For the distributed shipment strategy (Case 1), the total distances under perturbation are summarized in Table 2. The results show increasing deviations from the optimal solution as the perturbation increases, with significant distance increases for higher standard deviations.

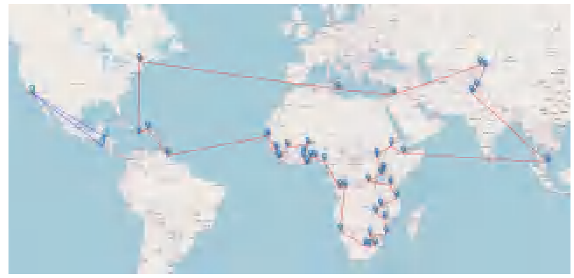
Standard Deviation (std)	Total Distance (miles)
1	38,098.6
2	38,514.9
5	41,148.5
10	47,365.3

Table 2: Sensitivity analysis for Problem of Optimizing Splitted Shipment - Case 1.

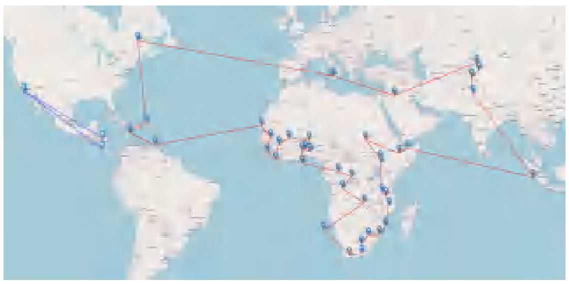
The following visualizations (Figure 5) illustrate the optimized routes under different levels of perturbation for Problem 2, Case 1.



(a) Standard deviation: 1



(b) Standard deviation: 2



(c) Standard deviation: 5

(d) Standard deviation: 10

Figure 5: Optimized routes under different perturbations for Problem of Optimizing Splitted Shipment - Case 1.

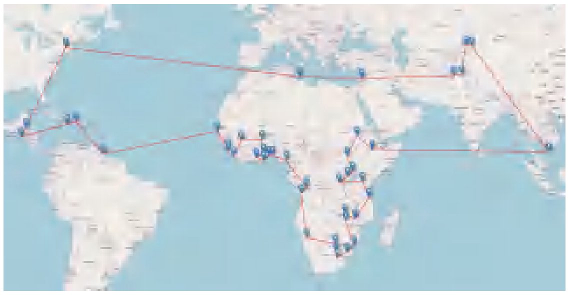
Case 2: Centralized Shipments

For the centralized shipment strategy (Case 2), the total distances under perturbation are summarized in Table 3. Compared to Case 1, the centralized strategy shows relatively smaller deviations at lower perturbation levels but still experiences significant distance increases for high perturbations.

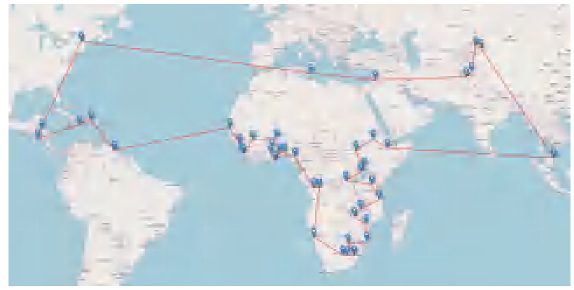
Standard Deviation (std)	Total Distance (miles)
1	35,033.2
2	35,214.0
5	37,498.5
10	42,001.8

Table 3: Sensitivity analysis for Problem of Optimizing Splitted Shipment - Case 2.

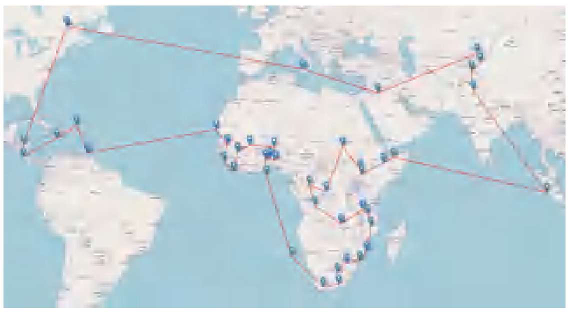
The following visualizations (Figure 6) illustrate the optimized routes under different levels of perturbation for Problem 2, Case 2.



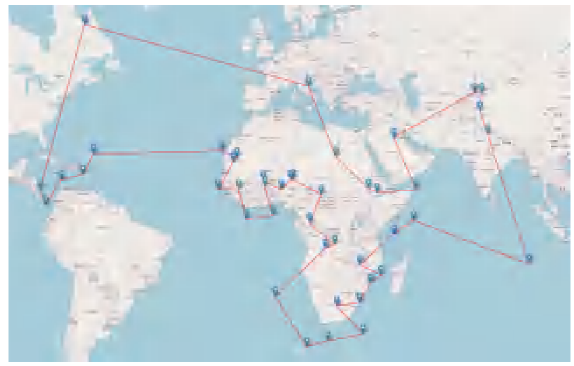
(a) Standard deviation: 1



(b) Standard deviation: 2



(c) Standard deviation: 5



(d) Standard deviation: 10

Figure 6: Optimized routes under different perturbations for Problem of Optimizing Splitted Shipment - Case 2.

4.3 Discussion

The sensitivity analysis reveals the following insights:

- As the perturbation level increases, the total distance tends to deviate significantly from the original optimal solution, particularly for high noise levels (std = 5 or 10).
- The centralized shipment strategy (Case 2) exhibits greater robustness to small perturbations compared to the distributed strategy (Case 1), likely due to its simpler structure and fewer dependencies on inter-hub coordination.
- The single consolidated route (Problem 1) is highly sensitive to large perturbations, as it requires precise alignment of geographically dispersed destinations to achieve optimal efficiency.

These results emphasize the importance of robust route optimization methodologies that account for potential variations in input data. Future work should explore dynamic re-optimization techniques to address such uncertainties in real-time.

5 Conclusion

This report addressed two critical optimization challenges within the U.S. government's Global Health Commodity Support Plan, focusing on enhancing the sustainability and efficiency of shipping essential health commodities to countries in need. The results underscore the importance of strategic route planning, rigorous constraint enforcement, and the impact of operational assumptions on logistics performance.

For the **Problem of Optimizing Shipment**, the objective was to design a single consolidated route minimizing the total travel distance. Key constraints, including start and return conditions at JFK and LAX, as well as subtour elimination via the Miller-Tucker-Zemlin (MTZ) formulation, were implemented to ensure feasibility. The optimization successfully achieved a total travel distance of **35,311 miles**, minimizing CO_2 emissions while ensuring all countries received their allocated health commodities in one comprehensive trip. The findings highlight that a single-route strategy is effective in scenarios where logistical simplicity and tight resource constraints are priorities.

For the **Problem of Optimizing Splitted Shipment**, two cases were analyzed to explore the trade-offs between distributed and centralized operations. The optimization objective, implemented using separate routing constraints for two flights originating from JFK and LAX, ensured no overlap of destinations. The following insights emerged:

- **Case 1:** Distributed shipments via two hubs (JFK and LAX) resulted in a total travel distance of **37,960 miles**. This approach reduced the environmental impact of each flight individually by leveraging geographical proximity but required higher coordination.

- **Case 2:** Centralized shipment operations with a single hub (JFK) achieved a slightly lower total travel distance of **35,116 miles**, simplifying logistics while still being close to the optimal distance of the consolidated route.

The use of MTZ constraints in both problems ensured route feasibility by eliminating subtours, while navigation and conservation constraints maintained operational consistency for each airport. These findings illustrate the flexibility and complexity required to optimize global supply chains for critical health commodities. Distributed shipment strategies, as seen in Case 1 of the second problem, offer operational efficiency for regional clusters, while centralized strategies, as seen in Case 2, balance simplicity and near-optimal distances.

In conclusion, the results emphasize the importance of strategic optimization in global health logistics. By leveraging rigorous constraints and optimization techniques, the U.S. government can enhance the effectiveness of its Global Health Commodity Support Plan. These methodologies not only ensure timely delivery of life-saving commodities but also align with broader goals of environmental sustainability and resource efficiency. Future work should consider integrating real-time data, dynamic demand forecasting, and multi-modal transportation options to further enhance supply chain adaptability and resilience.